Nama : Mohammad Febryan Khamim

NRP : 5002221072

6/5 2024

Tugas 4.2 Analisis Vektor (C)

Cari 6 soal - jawab terkait bab 4.2

1). Hitung $\int_C x^2 y \, ds$ dengan C ditentukan oleh parameter $x = 3\cos t$; $y = 3\sin t$; $0 \leq t \leq \frac{\pi}{2}$. Buktikan juga untuk parameter $x = \sqrt{9-y^2}$, $y = y$, $0 \leq y \leq 3$

Jawab :

•> Uji ketika parameter :

$$\int_a^b f(x(t), y(t)) \sqrt{[x'(t)]^2 + [y'(t)]^2} \, dt = \int_0^{\pi/2} x^2 y \sqrt{(-3\sin t)^2 + (3\cos t)^2} \, dt$$

$$= \int_0^{\pi/2} x^2 y \sqrt{9[\cos^2 t + \sin^2 t]} \, dt$$

$$= \int_0^{\pi/2} 9\cos^2 t \, (3\sin t) \cdot 3 \, dt$$

$$= 81 \int_0^{\pi/2} \sin t \cos^2 t \, dt$$

misal : $p = \cos t$ BB : $p = 1$

$dp = -\sin t \, dt$ BA : $p = 0$

$$= -81 \int_1^0 p^2 \cdot dp$$

$$= -81 \cdot \frac{1}{3} p^3 \Big]_1^0$$

$$= \frac{81}{3}$$

$$= 27 \,//$$

•> Ketika non parameter :

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy = \sqrt{1 + \left[\frac{1 \, (-\cancel{2}y)}{\cancel{2}\sqrt{9-y^2}}\right]^2} \, dy$$

$$= \sqrt{1 + \frac{y^2}{9-y^2}} \, dy$$

$$= \sqrt{\frac{9 - y^2 + y^2}{9-y^2}} \, dy$$

$$= \sqrt{\frac{9}{9-y^2}} \, dy$$

$$ds = \frac{3}{\sqrt{9-y^2}}\,dy$$

$$※ \int_C x^2y\,ds = \int_0^3 (9-y^2)\,y \cdot \frac{3}{(9-y^2)^{1/2}}\,dy$$

$$= 3\int_0^3 y\sqrt{9-y^2}\,dy$$

$※\ 9-y^2 = p$ BA : 0

$-2y\,dy = dp$ BB : 9

$$= 3 \cdot \int_9^0 -\frac{1}{2}\sqrt{p}\,dp$$

$$= -\frac{3}{2}\frac{2}{3}\,p\sqrt{p}\,\Big]_9^0$$

$$= 27$$

[Terbukti Sama]

2). Hitung integral garis $\int_C f\,ds$, dimana C adalah bagian atas lingkaran $r(s) = \langle \cos s, \sin s \rangle$ untuk $0 \le s \le \pi$ dan $f(x,y) = x+y$

$r'(s) = \langle -\sin s, \cos s \rangle$

$|r'(s)| = \sqrt{(-\sin s)^2+(\cos s)^2}$

$= \sqrt{\sin^2 s + \cos^2 s}$

$= 1$

$$\int_0^\pi f\,ds = \int_0^\pi (\cos s + \sin s)\sqrt{(-\sin s)^2+(\cos s)^2}\,ds$$

$$= \int_0^\pi (\cos s + \sin s)\,ds$$

$$= \sin s - \cos s\,\Big]_0^\pi$$

$$= 0 + 1 - (-1)$$

$$= 2$$

$\boxed{\text{Jawab} = 2}$

3). Hitunglah integral garis $\int_C f\,ds$, dimana C adalah segmen garis yang diberikan oleh $r(t) = (-2+6t,\ 5+8t)$ untuk $0 \le t \le 1$ dan $f(x,y) = x^2+y^2$

$r'(t) = (6, 8)$

$|r'(t)| = \sqrt{6^2+8^2}$

$= \sqrt{100}$

$= 10$

$$\int_0^1 f\,ds = \int_0^1 (-2+6t)^2 + (5+8t)^2\sqrt{6^2+8^2}\,dt$$

$$= \int_0^1 (36t^2 - 24t + 4 + 64t^2 + 80t + 25)\,10\,dt$$

$$= 10\int_0^1 100t^2 + 56t + 29\,dt$$

$$= 10\left[\frac{100}{3}t^3 + 28t^2 + 29t\,\Big]_0^1\right]$$

$$= 10 \cdot \left(29 + 28 + \frac{100}{3}\right)$$

$$= 10\left(\frac{87+84+100}{3}\right) = \frac{2710}{3}$$

$\boxed{\text{Jawab} = \frac{2710}{3}}$

4). Hitunglah $\int_c f\,ds$ with $f(x,y,z) = x^2 + y - 4z$ dengan C adalah ruas garis lurus dari $(4,3,1)$ hingga $(0,1,1)$.

Jawab :

Diketahui : $x = 4 - 4t$

$y = 3 - 2t$

$z = 1$

Pada $0 \leq t \leq 1$

$$\Rightarrow \int_0^1 \left[(4-4t)^2 + (3-2t) - 4\right]\left[\sqrt{4^2 + 2^2 + 0}\right] dt$$

$$= \int_0^1 \left[16 - 32t + 16t^2 + 3 - 2t - 4\right] 2\sqrt{5}\, dt$$

$$= 2\sqrt{5} \int_0^1 16t^2 - 34t + 15\, dt$$

$$= 2\sqrt{5} \left[\frac{16}{3}t^3 - \frac{34}{2}t^2 + 15t \Big]_0^1 \right]$$

$$= 2\sqrt{5} \left[\frac{16}{3} - \frac{34}{2} + 15\right]$$

$$= 2\sqrt{5} \left[\frac{32 - 102 + 90}{6}\right]$$

$$= \sqrt{5} \left[\frac{20}{3}\right]$$

$$= \frac{20}{3}\sqrt{5} \quad //$$

5). Tentukan panjang garis atau $\int_c f\,ds$ dengan $f(x,y,z) = x^2 + y^2 + z^2$ dengan $C(t) = (\underset{x}{\cos(t)}, \underset{y}{\sin(t)}, \underset{z}{t})$, $0 \leq t \leq \pi$

$$\int_c f(x,y,z)\,ds = \int_0^\pi (\cos^2 t + \sin^2 t + t^2)\sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2}\, dt$$

$$= \int_0^\pi (1 + t^2)\sqrt{2}\, dt$$

$$= \sqrt{2}\left[t + \frac{1}{3}t^3\Big]_0^\pi\right]$$

$$= \sqrt{2}\left[\pi + \frac{1}{3}\pi^3\right] \quad //$$

$$\boxed{\text{Jawab} = \sqrt{2}\left[\pi + \frac{\pi^3}{3}\right]}$$

//

6). $f(x,y) = \sqrt{1 + 9xy}$, $y = x^3$, $0 \le x \le 1$

$C(t) = (t, t^3)$, $0 \le t \le 1$

$$\int_C f(x,y)\, ds = \int_0^1 \sqrt{1 + 9t(t^3)}\ \sqrt{1 + (3t^2)^2}\ dt$$

$$= \int_0^1 1 + 9t^4\ dt$$

$$= \left[t + \frac{9}{5} t^5 \right]_0^1$$

$$= 1 + \frac{9}{5}$$

$$= \frac{14}{5} \ //$$